arXiv:hep-ph/0204273 v1 23 Apr 2002

Majorana Mass Zeroes from Triplet VEV without Majoron Problem

P.H. Frampton, M.C. Oh and T. Yoshikawa
Institute of Field Physics, Department of Physics and Astronomy, University of North Carolina, Chapel Hill, NC 27599-3255.

It is shown how to obtain recently-proposed two-zero Majorana mass textures in models with three Higgs triplets with small VEVs and a sufficiently massive triplet Majoron by using abelian discrete symmetries. It is briefly discussed how in SU(5) grand unification where the triplets occur in **15**'s the neutrino textures can be related to up- and down- quark mass textures.

PACS numbers:

I. INTRODUCTION

As is well-known, in the standard model (SM) with only left-handed neutrino fields ν_{iL}, the neutrinos are necessarily massless because a Majorana mass term $m_{ij}\nu^\alpha_{iL}\nu^\beta_{jL}\epsilon_{\alpha\beta}$ breaks $SU(2)$ gauge symmetry and is not renormalizable. Thus an extension of the theory is necessary to accommodate the experimental observations [1, 2] of non-vanishing neutrino mass[3].

Keeping only left-handed neutrino fields for simplicity, there is an important question of what is the most economical extension? One simple possibility is surely the Zee model[4] which adds a singlet charged scalar and results at one-loop order in a Majorana mass matrix with vanishing diagonal entries. However, this model is now strongly disfavored by the combination of SuperKamiokande and SNO data [5, 6].

More generally, it was argued in [7] that *any* Majorana matrix with three texture zeroes (including the Zee model) is strongly disfavored phenomenologically. At most two such texture zeroes are permitted and of the fifteen ways of assigning two zeroes to the six inequivalen mass matrix entries only seven (classified as A1, A2, B1, B2, B3, B4 and C in [7]) survive comparison with the SuperKamiokande and SNO data.

Here we make an attempt to incorporate these permissable textures into a model which contains the SM Higgs doublet (H) with $< H >\sim$ 100GeV and one or more Higgs triplets (T_k) (k = 1,2,...) at least some having small non-vanishing VEV. In doing this, we must first address the well-known problem[8] of a triplet Majoron associated with making a non-vanishing $< T >\neq 0$.

Next we incorporate the two-zero textures that are allowed phenomenologically.

Finally we discuss the interpretation in terms of $SU(5)$ unification where the triplet T_k fields appear in $\mathbf{15}_k$ representations.

II. TRIPLET HIGGS MODEL

The triplet Higgs has Yukawa couplings only to the Majorana neutrinos. To get a sufficiently small neutrino mass from these interactions the vacuum expectation value (VEV) of the triplet Higgs has to be also very small to avoid incredibly small Yukawa couplings. The model with such triplet Higgs has a potential difficulty that spontaneous breakdown of lepton number (L) will lead to a very light pseodoscalar Nambu-Goldstone boson $\mathcal{J}$ (Majoron) which does not agree with the experimental width of Z decay. Thus, our model must be such that this state $\mathcal{J}$ has mass greater than half the Z mass to avoid this Majoron problem.

For the triplet Higgs, $\mathbf{T} = (T^1, T^2, T^3)$, the Yukawa coupling to left-handed lepton doublet $L_L = (\nu, e^-)_L$ is

$$\begin{aligned}
\mathcal{L} &= -f\overline{L^c_L}\sigma^a T^a L_L \\
&= -f\overline{e^c}(T^1 - iT^2)e + f\overline{\nu^c}(T^1 + iT^2)\nu \\
&\quad -f\overline{\nu^c}T^3 e - f\overline{e^c}T^3\nu \qquad (1)\\
&= -\sqrt{2}f\overline{e^c}T^{++}e + \sqrt{2}f\overline{\nu^c}T^0\nu \\
&\quad -f\overline{\nu^c}T^+ e - f\overline{e^c}T^+\nu. \qquad (2)
\end{aligned}$$

where

$$\begin{aligned}
T^{++} &= \frac{1}{\sqrt{2}}(T^1 - iT^2), \\
T^+ &= T^3, \\
T^0 &= \frac{1}{\sqrt{2}}(T^1 + iT^2).
\end{aligned}$$

The kinetic term of the triplet Higgs is

$$\left|\partial_\mu T - \frac{i}{2}\begin{pmatrix} \sqrt{g^2+g'^2}A_\mu & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & \sqrt{g^2+g'^2}Z_\mu \end{pmatrix} T\right|^2, \qquad (3)$$

where

$$\begin{aligned}
T &\equiv \sigma^a T^a \\
&= \begin{pmatrix} T^+ & \sqrt{2}T^{++} \\ \sqrt{2}T^0 & -T^+ \end{pmatrix}. \qquad (4)
\end{aligned}$$

The $Z-T-T$ couplings are

$$i\sqrt{g^2+g'^2}[T^{0*}Z^\mu\partial_\mu T^0-\partial_\mu T^{0*}Z^\mu T^0]. \qquad (5)$$

In this Majoron model, to give the tiny majorana neutrino mass T^0 has to get a tiny VEV. The magnitude may be $\sqrt{\Delta_a}\sim O(0.1)eV$ if the Yukawa coupling is on order unity. Then, we can decompose the neutral Higgs,

$$T^0=u+\frac{1}{\sqrt{2}}(\rho+i\mathcal{J}), \qquad (6)$$

where u is the VEV, ρ is real part and $\mathcal{J}$ is the imaginary part. From eq.(5), the $Z-T-T$ couplings are

$$\frac{1}{2}\sqrt{g^2+g'^2}Z^\mu[(\partial_\mu\rho)\mathcal{J}-\rho\partial_\mu\mathcal{J}]. \qquad (7)$$

So on the Z decay, it is $\sqrt{g^2+g'^2}m_z$. In this case, Z boson can decay to ρ and $\mathcal{J}$ because these masses are on order of u. If so, the ratio of the partial decay widths

$$\frac{\Gamma(Z\to\rho\mathcal{J})}{\Gamma(Z\to\nu\nu)}=2, \qquad (8)$$

would mean an additional invisible width of Z. This is inconsistent with the experimental data where the invisible width correspond quite precisely to that expected for three active neutrinos and leaves no room for triplet Majoron decay.

To avoid this difficulty, we therefore need to extend the model. As methods to solve this difficulty, we can consider three cases.

1) By Higgs mechanism, ρ and $\mathcal{J}$ have to be absorbed.
2) Giving a large mass to the Majorons without VEV.
3) Giving a large mass to the Majorons with tiny VEV.

The first case will generally produce unacceptably-light gauge bosons.

In the second case, we may consider a scenario in which neutrino masss arises from the radiative corrections. However, to permit a diagram contributing to Majorana neutrino masses we need a trilinear coupling $\mu\phi T\tilde{\phi}$, where ϕ is the doublet higgs. When electroweak symmetry is broken, ϕ has a VEV and induces a shift in the VEV of T as shown in Fig. **1**, because the trilinear coupling contributes to a linear term, and the small VEV for T is thus destabilized.

Hence only scenario (3) is viable, and it requires that T has a very small VEV while $\mathcal{J}$, and ρ, have heavy masses $M_{\mathcal{J},\rho}>M(Z)/2$.

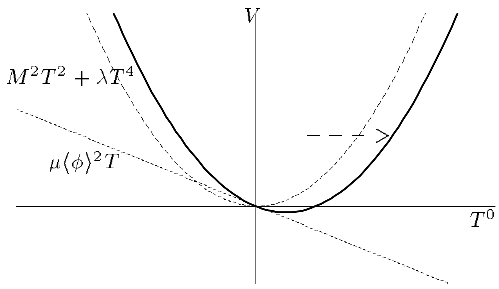

Fig. 1

This third case which we employ has been discussed by Ma and Sarkar[9]. They induce the large mass of ρ and $\mathcal{J}$ by adding lepton number violating terms in the Higgs potential.

The most general Higgs potential of a doublet Higgs $\Phi=(\phi^+,\phi^0)$ and a triplet Higgs $T=(T^{++},T^+,T^0)$ is

$$\begin{aligned} V &= m^2\phi^\dagger\phi+M^2T^\dagger T \\ &+ \frac{1}{2}\lambda_1(\phi^\dagger\phi)^2+\frac{1}{2}\lambda_2(T^\dagger T)^2 \\ &+ \lambda_3(\phi^\dagger\phi)(T^\dagger T) \\ &- \lambda_4(i\varepsilon_{ijk}\phi^\dagger\sigma^i\phi T^{\dagger j}T^k)+h.c. \\ &+ \mu\phi^\dagger T^a\sigma^a\tilde{\phi}+h.c.. \end{aligned} \qquad (9)$$

The important term is that with coupling μ. Decomposing it, the terms are

$$\mu\sqrt{2}[\phi^0\phi^0T^{0*}+\sqrt{2}\phi^+\phi^0T^--\phi^+\phi^+T^{--}+h.c.] \qquad (10)$$

The potential for neutral Higgses is

$$\begin{aligned} & m^2\phi^{0*}\phi^0+M^2T^{0*}T^0+\frac{\lambda_1}{2}(\phi^{0*}\phi^0)^2 \\ +\ & \frac{\lambda_2}{2}(T^{0*}T^0)^2+(\lambda_3+\lambda_4)(\phi^{0*}\phi^0)(T^{0*}T^0) \\ +\ & \sqrt{2}\mu(\phi^0\phi^0T^{0*}+\phi^{0*}\phi^{0*}T^0) \end{aligned} \qquad (11)$$

In this case, the conditions for stationarizing VEV are

$$m^2+\lambda_1v^2+\lambda_3u^2+\lambda_4u^2+2\sqrt{2}\mu u=0 \quad (12)$$

$$M^2u+\lambda_2u^3+\lambda_3v^2u+\lambda_4v^2u+\sqrt{2}\mu v^2=0 \quad (13)$$

The neutral Higgses are decomposed as follows:

$$T^0 = u+\frac{1}{\sqrt{2}}(\rho+i\mathcal{J}) \qquad (14)$$

$$\phi^0 = v+\frac{1}{\sqrt{2}}(\xi_1+i\xi_2) \qquad (15)$$

To get the neutrino mass from the tiny VEV of the triplet Higgs, we need a hierarchy $M, \mu \gg v \gg u$. When we assume the couplings λ_i are parameters of order one, then, from eq. (13),

$$u \sim -\frac{\sqrt{2}\mu v^2}{M^2} \tag{16}$$

If we take $u \sim O(10^{-1})$ GeV, we need $M \sim 10^{14} GeV$ and μ is also order M or less. The mixing terms between the doublet one and the triplet one will appear by λ_4 and μ terms. The mass matrices for (ξ_1, ρ) and $(\xi_2, \mathcal{J})$ are respectively

$$\begin{pmatrix} 2\lambda_1 v^2 & 2(\lambda_3+\lambda_4)uv + 2\sqrt{2}\mu v \\ 2(\lambda_3+\lambda_4)uv + 2\sqrt{2}\mu v & 2\lambda_2 u^2 - \sqrt{2}\mu\frac{v^2}{u} \end{pmatrix} \tag{17}$$

and

$$\begin{pmatrix} -4\sqrt{2}\mu u & 2\sqrt{2}\mu v \\ 2\sqrt{2}\mu v & -\sqrt{2}\mu\frac{v^2}{u} \end{pmatrix} \tag{18}$$

Thus, the mass eigenvalues are

$$(\xi_1^m, \rho^m) \sim (2\lambda_1 v^2, -\sqrt{2}\mu\frac{v^2}{u}) \tag{19}$$

$$(\xi_2^m, \mathcal{J}^m) = (0, -\sqrt{2}\mu\frac{4u^2+v^2}{u}) \tag{20}$$

The would-be Nambu-Goldstone boson ξ_2 is eaten by Z and the would-be Majoron $\mathcal{J}^m$ gets a large mass. Because of the smallness of u, the mass of $\mathcal{J}^m$ will be much larger than M_W if the parameter μ is not so small value. Hence, in this case, Z boson kinematically cannot decay to $\rho\mathcal{J}$ final state.

The charged scalars also get the large mass. The masses of T^{+m} and T^{++} are proportional to $\mu\frac{v^2}{u}$.

In this model, the radiative mass correction is

$$\delta m_\nu \sim f\frac{m_l^2\mu}{(4\pi)^2 M^2} \tag{21}$$

where M is the charged triplet mass and about $10^{14} GeV$. This is at most 10^{-6} eV and so cannot be the main contribution to the neutrino mass, as it is too small to explain the LMA solution.

III. INCORPORATION OF TWO-ZERO TEXTURES.

In [7] it was pointed out that at most two of the six independent elements of the symmetric Majorana mass matrix can vanish and the phenomenologically-allowed possibilities were classified as A1, A2, B1, B2, B3, B4, and C respectively. It is therefore of interest to accommodate these textures in our triplet Higgs model. To do this, we impose on our model various discrete Z_p symmetries which give rise to these two-zero textures in a technically-natural manner.

The most realistic Majorana neutrino mass matrix to explain the experiments for solar and atmospheric neutrinos is

$$\begin{pmatrix} \delta & m_1 & m_2 \\ m_1 & \epsilon_1 & \epsilon_2 \\ m_2 & \epsilon_2 & \epsilon_3 \end{pmatrix}. \tag{22}$$

Here we need the mass hierarchy that is $m_1 \sim m_2 \gg \delta, \epsilon_i$ [5, 11]. This is corresponds to Case C of the possible zeros textures in Ref.[7] if $\epsilon_1 = \epsilon_3 = 0$. For Case C [7] we need at least three triplet Higgs T_k (k = 1,2,3) and we asign them under Z_3 as following, $\nu_1, \nu_2, \nu_3 \to \nu_1, \omega\nu_2, \omega^2\nu_3$ and $T_1, T_2, T_3 \to T_1, \omega T_2, \omega^2 T_3$, where $\omega = \sqrt[3]{1}$ is a generator of Z_3. Then the Majorana neutrino mass matrix is

$$\begin{pmatrix} f_{11}\langle T_1\rangle & f_{12}\langle T_3\rangle & f_{13}\langle T_2\rangle \\ f_{12}\langle T_3\rangle & f_{22}\langle T_2\rangle & f_{23}\langle T_1\rangle \\ f_{13}\langle T_2\rangle & f_{23}\langle T_1\rangle & f_{33}\langle T_3\rangle \end{pmatrix}, \tag{23}$$

where $\langle T_i\rangle$ is the VEV of the neutral triplet Higgs. Under a further Z_2, if $\nu_1 \to -\nu_1$, $\nu_2, \nu_3 \to \nu_2, \nu_3$, $T_1 \to T_1$ and $T_2, T_3 \to -T_2, -T_3$, The elements of {22} and {33} will disappear. And by taking $\langle T_2\rangle \sim \langle T_3\rangle > \langle T_1\rangle$, we can get most realistic mass matrix which is the C type texture.

$$\begin{pmatrix} f_{11}\langle T_1\rangle & f_{12}\langle T_3\rangle & f_{13}\langle T_2\rangle \\ f_{12}\langle T_3\rangle & 0 & f_{23}\langle T_1\rangle \\ f_{13}\langle T_2\rangle & f_{23}\langle T_1\rangle & 0 \end{pmatrix}. \tag{24}$$

On the other hand, we can make also the Case $B1$ and Case $B2$. By taking $\langle T_2\rangle = 0$ without Z_2 symmetry,

$$\begin{pmatrix} f_{11}\langle T_1\rangle & f_{12}\langle T_3\rangle & 0 \\ f_{12}\langle T_3\rangle & 0 & f_{23}\langle T_1\rangle \\ 0 & f_{23}\langle T_1\rangle & f_{33}\langle T_3\rangle \end{pmatrix}. \tag{25}$$

For $\langle T_3\rangle = 0$,

$$\begin{pmatrix} f_{11}\langle T_1\rangle & 0 & f_{13}\langle T_2\rangle \\ 0 & f_{22}\langle T_2\rangle & f_{23}\langle T_1\rangle \\ f_{13}\langle T_2\rangle & f_{23}\langle T_1\rangle & 0 \end{pmatrix}. \tag{26}$$

Proceeding along these lines, we can accommodate all the two-zero textures of [7] as indicated in

	ν_1	ν_2	ν_3	T_1	T_2	T_3	
$Z_3 \times Z_2$	(1,-1)	$(\omega, 1)$	$(\omega^2, 1)$	(1,1)	$(\omega$, -1)	$(\omega^2$, -1)	C
Z_4	1	i	-i	1	i	-i	C
$Z_4 \times Z_2$	(1,1)	(i, -1)	(-i, 1)	(1,-1)	(-1, 1)	(i, 1)	A_1
$Z_4 \times Z_2$	(1,1)	(-i, 1)	(i, -1)	(1,-1)	(-1, 1)	(i, 1)	A_2
Z_3	1	ω	ω^2	1	$\times$	ω^2	B_1
Z_3	1	ω	ω^2	$\times$	ω	ω^2	B_2
$Z_4 \times Z_2$	(i,-1)	(1, 1)	(-i, 1)	(1,-1)	(-1, 1)	(i, 1)	B_3
$Z_4 \times Z_2$	(i,-1)	(-i, 1)	(1, 1)	(1,-1)	(-1, 1)	(i, 1)	B_4

TABLE I: Accommodating two-zero textures of [7].

the above Table where the classification of Majorana neutrino mass matrices with two zeros in [7] is used in the final column.

The triplet T occurs naturally in the **15** of SU(5). Although minmal $SU(5)$ is excluded, simple generalizations are consistent with experimental constraints[12]. Under $SU(5) \supset (SU(3)_C \times SU(2)_L)_Y$ the various SU(5) irreps decompose as

$$\begin{aligned} \mathbf{5} &\supset (3,1)_{-2/3} + (1,2)_{+1} \quad \text{(definition)} \\ \mathbf{15} \equiv (\mathbf{5} \times \mathbf{5})_s &\supset (6,1)_{-4/3} + (1,3)_{+2} + (3,2)_{+1/3} \\ \bar{\mathbf{15}} \equiv (\bar{\mathbf{5}} \times \bar{\mathbf{5}})_s &\supset (\bar{6},1)_{+4/3} + (1,3)_{-2} + (\bar{3},2)_{-1/3} \end{aligned}$$

and the T triplet may be identified with the $(1,3)_{\pm 2}$ occurring in $\mathbf{15}, \bar{\mathbf{15}}$.

Let us give one example of how the neutrino Majorana mass texture may be correlated with up- and down- quark mass textures of the types discussed in *e.g.* [13].

Take the example in the above table with symmetry $Z_4 \times Z_2$ which gives the neutrino mass texture designated A2. By subsuming this in $SU(5)$ with three **15**'s and requiring the three **10**'s of fermions to transform as $(i,-1), (1,-1), (-1,1)$ and adding five **5**'s of Higgs transforming as $(1,-1), (-i,-1), (i,1), (-i,1), (1,1)$. gives one of the five permissable five-zero quark mass textures in [13].

We hope to return to a more detailed analysis of this neutrino-quark linkage in a future publication.

Acknowledgments

This work was supported in part by the US Department of Energy under Grant No. DE-FG02-97ER-41036.

[1] S. Fukuda *et al.*, Super-Kamiokande collaboration, Phys. Rev. Lett. **81**, 1562 (1998);
S. Fukuda *et al.*, Super-Kamiokande collaboration, Phys. Rev. Lett. **86**, 5651 (2001);
S. Fukuda *et al.*, Super-Kamiokande collaboration, Phys. Rev. Lett. **86**, 5656 (2001).

[2] Q.R. Ahmad *et al.*, SNO collaboration, Phys. Rev. Lett. **87**, 071301 (2001). See also the recent data on neutral currents from SNO collaboration in: `nucl-ex/0204008` and `nucl-ex/0204009`

[3] S.M. Barr, in Proceedings of the Eighth International Symposium on Particles, Strings and Cosmology (PASCOS 2001). Editors: P.H. Frampton and Y.J.Ng. Rinton Publishing (2001) page 243.
I. Dorsner and S.M. Barr, Nucl. Phys. **B617**, 493 (2001).

[4] A. Zee, Phys. Lett.**B93**, 389,(1980).
P.H. Frampton and S.L. Glashow, Phys. Lett. **B461,** 95 (1999).

[5] P.H. Frampton, M.C. Oh and T. Yoshikawa, Phys. Rev. **D65**, 073014 (2002).

[6] Y. Koide, Phys. Rev. **D64** 077301 (2001).

[7] P.H. Frampton, S.L. Glashow and D. Marfatia, hep-ph/0201008. Phys. Lett. **B** (in press).

[8] G.B. Gelmini and M. Roncadelli, Phys. Lett. **B99**, 411 (1981),
G.B. Gelmini and E. Roulet, Rept. Prog. Phys. **58**, 1207 (1995) .

[9] E. Ma and U. Sarkar, Phys. Rev. Lett. **80**, 5716 (1998).

[10] T. Kitabayashi and M. Yasue, Phys. Lett. **B524**, 308 (2002);
T. Kitabayashi and M. Yasue, hep-ph/0112287.

[11] K.S. Babu and R.N. Mohapatra, hep-ph/0201176.

[12] P.H. Frampton and S.L. Glashow, Phys. Lett. **131B,** 340 (1983). U. Amaldi, W. De Boer, P.H. Frampton, H. Furstenau and J.T. Liu, Phys. Lett. **281B,** 374 (1992)

[13] P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. **B406,** 19 (1993).
P.H. Frampton and O.C.W. Kong, Phys. Rev. Lett. **75,** 781 (1995).